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LETTER TO THE EDITOR

On the moving A–B phase boundary of superfluid ^{3}He

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Abstract. We investigate the A–B phase boundary of superfluid ^{3}He at very low temperatures under near-equilibrium conditions where it expands with uniform velocity v into the undercooled A phase. Neglecting collisions between excitations, essentially exact solutions to the 'microscopic' Gorkov equations are found that describe fermionic excitations which scatter from the moving interface. The distribution function of such fermions is characterized by an essential singularity at $v = 0$ and translates rigidly with the A–B phase boundary only below a critical speed v_{c1}.

When normal ^{3}He is liquified and cooled at pressures P above 21 bar and temperatures $T_c(P) \approx 2$ to 2.5 mK, it undergoes a second-order phase transition into the superfluid A phase. On further cooling, the A phase undergoes a first-order phase transition at a temperature $T_{AB}(P)$ into the B phase [1]. The first-order character of this phase transition implies that when the A and B phases coexist, the phase boundary that forms between them is of finite thickness—resulting in an order parameter profile which has a rapid change of symmetry [2]. It is an experimental fact that the A phase may be substantially supercooled below $T_{AB}(P)$ and as a result the nucleated B phase will expand into the A phase [3].

It might appear that there is little to observe of the A–B phase boundary since the density difference between bulk A and B phases is very small. This viewpoint, however, misses the subtle reflection and transmission processes of quasiparticles and Cooper pairs which can occur at sharp spatial structures in the order parameter [4]. For example, an incident quasiparticle from the A phase with energy below the bulk B-phase gap cannot be transmitted into the B phase as a propagating quasiparticle excitation. There is, however, the possibility for the incident quasiparticle to acquire a partner at the interface to form a Cooper pair, which then propagates into the B phase. The quasihole created by this process propagates back into the A phase. Effectively a branch conversion has occurred where incident quasiparticles are retroreflected as quasiholes (Andreev reflection [5]).

It is important to note that the acquisition of a partner in the above description of Andreev scattering occurs in a non-unique manner at the A–B phase boundary. An incident quasiparticle from the A phase with, say, spin ↑ may catch a partner with either spin ↑ or ↓ to form a Cooper pair in the B phase, leaving behind a correspondingly spin-polarized quasihole. The outcome of this is that the A–B

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phase boundary is capable of changing the polarization of scattered excitations [6]. In the absence of particle–hole asymmetry, however, these effects lead to no net equilibrium magnetization [6].

In the following we present the first attempts towards a microscopic theory of the moving A–B phase boundary that fully incorporates the above mentioned branch conversion processes (including over-the-barrier reflection), and show the existence of a hitherto unsuspected critical velocity v_{cl} for a rigidly translating interface that interpolates from the A phase (far to the left) to the B phase (far to the right). We assume [7]

(i) the A–B interface translates rigidly along the x axis with velocity $\dot{\boldsymbol{R}}_0(t) = \dot{X}_0(t)\hat{\boldsymbol{x}}$,

(ii) coupling to order parameter collective modes may be neglected (low excitation density),

(iii) the spin-$\frac{1}{2}$ excitations of the superfluid move ballistically (collisionless limit),

(iv) there is no temperature difference between the A and B sides.

In the bulk, far on either side from the moving kink in the order parameter, there are no mass currents due to the absence of spatially varying phases [4]. In general, the full order parameter $\Delta(\boldsymbol{R}-\boldsymbol{R}_0(t), t; \hat{\boldsymbol{p}})$ will not simply have a rigidly translating form but will include corrections that fall into two classes. The first class, of a more technical nature, deals with smoothing corrections to a piecewise constant *ansatz* [6]. While a self-consistent solution will certainly be smooth in the interfacial region, we believe that a piecewise constant profile is the correct 'zeroth-order' approximation to the problem [8]. The second class of corrections is more fundamental and is concerned with the stability of a rigidly moving A–B phase boundary and the dynamics of the pair condensate [14] in the interfacial region (collective modes). Taking for granted the stability of a rigidly moving interface and assuming there is no net energy transfer from the order-parameter degrees of freedom to the excitations in the ^{3}He superfluid, it is interesting to speculate whether propagation without a dissipative friction force is possible in the ballistic limit. The point of view of this paper is that the order parameter can translate rigidly and we shall investigate the consequences of this concept. In particular we determine the distribution function (time-ordered Green's function) for quasiparticles by solving the Gorkov equations for the Wigner-transformed Green's functions [9, 10]

$$\begin{pmatrix} \epsilon - \xi(\boldsymbol{p}) & \Delta(\boldsymbol{R}-\boldsymbol{R}_0(t), t; \hat{\boldsymbol{p}}) \cdot \boldsymbol{\sigma} \\ \Delta^*(\boldsymbol{R}-\boldsymbol{R}_0(t), t; \hat{\boldsymbol{p}}) \cdot \boldsymbol{\sigma} & \epsilon + \xi(\boldsymbol{p}) \end{pmatrix} \otimes G_{\rm w}(\boldsymbol{R}, t; \boldsymbol{p}, \epsilon) = \hat{\tau}_0. \qquad (1)$$

In general, $\boldsymbol{R}_0(t)$ denotes the position of an arbitrarily moving structure in the order parameter, e.g. a point defect, a line defect or a planar phase boundary. The vector $\boldsymbol{R}$ denotes the position, t the time, $\boldsymbol{p}$ the momentum and ϵ the energy associated with the quasiparticles in the laboratory frame. Additionally, $\xi(\boldsymbol{p}) = p^2/2m - \mu$ is the free-quasiparticle kinetic energy, relative to the chemical potential μ. The $\otimes$ produce between Wigner-transformed functions [9] in equation (1) couples the time dependence of the collective coordinate $\boldsymbol{R}_0(t)$ in the order parameter with the energy argument in the Green's function.

A Galilean transformation *combined* with a gauge transformation can be used to eliminate $\boldsymbol{R}_0(t)$ from the problem. We make the following *ansatz* that applies to the retarded, advanced and time-ordered Green's functions:

$$G_{\rm w}(\boldsymbol{R},t;\boldsymbol{p},\epsilon)=\hat{\tau}_3\Gamma\big(\boldsymbol{R}-\boldsymbol{R}_0(t),t;\boldsymbol{p},\epsilon-\boldsymbol{p}\cdot\dot{\boldsymbol{R}}_0(t)\big) \tag{2}$$

where $\dot{\boldsymbol{R}}_0(t)$ specifies the instantaneous velocity of the moving structure in the order parameter relative to the laboratory frame. Under the coordinate transformation

$$\boldsymbol{R}'=\boldsymbol{R}-\boldsymbol{R}_0(t)\qquad t'=t\qquad \boldsymbol{p}'=\boldsymbol{p}\qquad \epsilon'=\epsilon-\boldsymbol{p}\cdot\dot{\boldsymbol{R}}_0(t)$$

the derivative operators transform according to

$$\vec{\partial}_{\boldsymbol{R}}=\vec{\partial}_{\boldsymbol{R}'}\qquad \partial_t=\partial_{t'}-\dot{\boldsymbol{R}}_0\cdot\vec{\partial}_{\boldsymbol{R}'}-\boldsymbol{p}\cdot\ddot{\boldsymbol{R}}_0\partial_{\epsilon'}\qquad \vec{\partial}_{\boldsymbol{p}}=\vec{\partial}_{\boldsymbol{p}'}-\dot{\boldsymbol{R}}_0\partial_{\epsilon'}\qquad \partial_\epsilon=\partial_{\epsilon'}$$

leaving the generator of the $\otimes$ product [9] invariant.

The equation of motion for the propagator $\Gamma(\boldsymbol{R}',t';\boldsymbol{p}',\epsilon')$ then follows by straightforwardly inserting equation (2) into equation (1)

$$\begin{pmatrix}(\epsilon'+\boldsymbol{p}'\cdot\dot{\boldsymbol{R}}_0)-\xi(\boldsymbol{p}') & -\boldsymbol{\Delta}(\boldsymbol{R}',t';\hat{\boldsymbol{p}}')\cdot\boldsymbol{\sigma}\\ \boldsymbol{\Delta}^*(\boldsymbol{R}',t';\hat{\boldsymbol{p}}')\cdot\boldsymbol{\sigma} & -(\epsilon'+\boldsymbol{p}'\cdot\dot{\boldsymbol{R}}_0)-\xi(\boldsymbol{p}')\end{pmatrix}\otimes\Gamma(\boldsymbol{R}',t';\boldsymbol{p}',\epsilon')=\hat{\tau}_0. \tag{3}$$

In what follows we neglect higher-order corrections of order $1/\xi_{\rm BCS}p_{\rm F}$ in the expansion implied by the $\otimes$ product [9], and we work in *dimensionless scaled units* [11]. For a planar interface with surface normal $\hat{\boldsymbol{x}}$ moving at *constant* velocity $\dot{\boldsymbol{R}}_0(t)=\boldsymbol{v}=-v\hat{\boldsymbol{x}}$ the quantities in equation (3) are independent of y' and z' and taken to be independent of t'. We solve equation (3) subject to the boundary condition of vanishing x'-derivatives as $|x'|\to\infty$. For the stratified-medium *ansatz* [7], $\boldsymbol{\Delta}(\boldsymbol{R}';\hat{\boldsymbol{p}})=\boldsymbol{\Delta}_{\rm A}(\hat{\boldsymbol{p}})\theta(-x')+\boldsymbol{\Delta}_{\rm B}(\hat{\boldsymbol{p}})\theta(x')$, the solution follows simply by matching plane waves and may be represented in the following compact form [12]:

$$\Gamma(x;\boldsymbol{p},\epsilon)=\mathsf{W}_{\rm L}^{\rm T}\left\{\begin{array}{l}\left[\frac{1}{\mathsf{M}_{\rm B}}+\exp(2\mathrm{i}x\mathsf{M}_{\rm B})\mathsf{P}_{\rm B}^{+}\cdot\frac{1}{\mathsf{P}_{\rm B}^{+}+\mathsf{P}_{\rm A}^{-}}\left(\frac{1}{\mathsf{M}_{\rm A}}-\frac{1}{\mathsf{M}_{\rm B}}\right)\right]\theta(x)\\ +\left[\frac{1}{\mathsf{M}_{\rm A}}-\exp(2\mathrm{i}x\mathsf{M}_{\rm A})\mathsf{P}_{\rm A}^{-}\cdot\frac{1}{\mathsf{P}_{\rm B}^{+}+\mathsf{P}_{\rm A}^{-}}\left(\frac{1}{\mathsf{M}_{\rm A}}-\frac{1}{\mathsf{M}_{\rm B}}\right)\right]\theta(-x)\end{array}\right\}\mathsf{W}_{\rm R}. \tag{4}$$

For notational simplicity we shall from now on omit the primes on symbols, there being no danger of confusion with the laboratory frame: $\mathsf{M}_{\rm A}(v,\boldsymbol{p},\epsilon)$ and $\mathsf{M}_{\rm B}(v,\boldsymbol{p},\epsilon)$ denote the bulk limits of the 8×8 matrix $\mathsf{M}(x;v,\boldsymbol{p},\epsilon)$ on the A and the B side, while $\mathsf{W}_{\rm L}^{\rm T}$ and $\mathsf{W}_{\rm R}$ are 4×8 and 8×4 *rectangular* matrices, respectively. In 4×4 block-matrix notation they read

$$\mathsf{M}(x;v,\boldsymbol{p},\epsilon)=\begin{pmatrix}-\mathsf{a} & \hat{\tau}_0\\ \mathsf{H}(x;v,\boldsymbol{p},\epsilon) & -\mathsf{a}\end{pmatrix} \tag{5a}$$

$$\mathsf{H}(x;v,\boldsymbol{p},\epsilon)=\begin{pmatrix}\mu+v^2-p_y^2-p_z^2+\epsilon & -\boldsymbol{\Delta}(x,\hat{\boldsymbol{p}})\cdot\boldsymbol{\sigma}\\ \boldsymbol{\Delta}^*(x,\hat{\boldsymbol{p}})\cdot\boldsymbol{\sigma} & \mu+v^2-p_y^2-p_z^2-\epsilon\end{pmatrix} \tag{5b}$$

$$\mathsf{a}=p_x\hat{\tau}_0+v\hat{\tau}_3 \qquad \mathsf{W}_{\mathrm{L}}^{\mathrm{T}}=(\hat{\tau}_0,\mathsf{O}) \qquad \mathsf{W}_{\mathrm{R}}=\begin{pmatrix}\mathsf{O}\\ \hat{\tau}_0\end{pmatrix}. \tag{5c}$$

The other matrices, $\mathsf{P}_{\mathrm{A}}^{-}$ and $\mathsf{P}_{\mathrm{B}}^{+}$, are projection operators which serve to suppress the exponential growth of $\Gamma(x;\boldsymbol{p},\epsilon)$ as x approaches the bulk A and B phases. They have a simple representation in terms of complex contour integrals:

$$\mathsf{P}_{\mathrm{B}}^{+}=\int_{\mathrm{C}_{\mathrm{B}}^{+}}\frac{\mathrm{d}z}{2\pi\mathrm{i}}\frac{1}{z-\mathsf{M}_{\mathrm{B}}} \qquad \mathsf{P}_{\mathrm{A}}^{-}=\int_{\mathrm{C}_{\mathrm{A}}^{-}}\frac{\mathrm{d}z}{2\pi\mathrm{i}}\frac{1}{z-\mathsf{M}_{\mathrm{A}}}. \tag{6}$$

Here $\mathrm{C}_{\mathrm{B}}^{+}$ is a path enclosing all eigenvalues of M_{B} with positive imaginary part and $\mathrm{C}_{\mathrm{A}}^{-}$ is a path enclosing all eigenvalues of M_{A} with negative imaginary part. Note that the inversion of 8×8 matrices such as $z-\mathsf{M}$ can be performed analytically [13].

The retarded and advanced Green's functions $G_{\mathrm{W}}^{\mathrm{ret}}$ and $G_{\mathrm{W}}^{\mathrm{adv}}$ determine the local density of states of the fermions in the laboratory frame. These quantities approach their known *velocity-independent* bulk form far on either side from the interface. The x-dependent profile of the fermion distribution is determined from the requirement that, at a given instant, all the spin-$\frac{1}{2}$ quasiparticles *incident* onto the interface are distributed according to the asymptotic thermal equilibrium distribution. The latter has, after transformation to the moving frame, Doppler-shift arguments [16]. It turns out—perhaps surprisingly—that the problem of determining the finite-temperature, time-ordered Green's function for quasiparticles moving in the presence of a *rigidly* translating domain wall propagating at constant velocity v may be solved exactly

$$\Gamma(\boldsymbol{R};\boldsymbol{p},\epsilon)=\Gamma^{\mathrm{adv}}(\boldsymbol{R};\boldsymbol{p},\epsilon)\otimes \mathrm{f}(\epsilon+\boldsymbol{v}\dot{\boldsymbol{p}})+\Gamma^{\mathrm{ret}}(\boldsymbol{R};\boldsymbol{p},\epsilon)\otimes \mathrm{f}(-\epsilon-\boldsymbol{v}\dot{\boldsymbol{p}}) \tag{7}$$

with $\mathrm{f}(\epsilon)=1/[1+\exp(\epsilon/T)]$ the Fermi–Dirac distribution at temperature T.

Note that the $\otimes$ operator leads here to an intrinsic coupling of the spatial variation of Γ^{ret} and Γ^{adv} with the extra momentum dependence (Doppler shift) in the argument of the Fermi–Dirac distribution. The explicit form for the time-ordered Green's function follows straightforwardly from equations (2), (4) and (7).

It is evident from the functional form of equation (7) that we could have imposed the condition that, far ahead of the moving interface, we have the equilibrium bulk A phase at, say, temperature T. There, apparently, the time-ordered Green's function respects the Kubo–Martin–Schwinger condition [15]. Solving equation (3) with this boundary condition one finds that the time-ordered Green's function on the other side of the interface, deep in the B phase, relaxes back to the functional form of the thermal-equilibrium Green's function, despite the fact that all incident quasiparticles were scattered from the *moving* interface.

Equation (7) contains information about the stability of a *rigidly* translating A–B interface, though in highly coded form. Using the Fourier representation of the Heaviside step functions $\theta(\pm x)$ in equation (4) it may be shown that the non-local character of the $\otimes$ operator results in the 'dragging' of information from the A side onto the B side and vice versa. Thus, for example, the following $\otimes$ product, which appears in equation (7), takes for $x > 0$ the form [11]

$$[\exp(2\mathrm{i}xk_{\mathrm{A}})\theta(-x)] \otimes \mathrm{f}(\epsilon - 2vp_x) = T\sum_{\omega_n}\theta(\omega_n - 2v\ \mathrm{Im}\ k_{\mathrm{A}})\frac{\exp\{\mathrm{i}x[2p_x + (\mathrm{i}\omega_n - \epsilon)/v]\}}{\mathrm{i}\omega_n - \epsilon + 2v(p_x - k_{\mathrm{A}})} \tag{8}$$

where $\mathrm{i}\omega_n = \mathrm{i}(2n_1)\pi T$ denotes poles of the Fermi–Dirac distribution at temperature T and $k_{\mathrm{A}} = k_{\mathrm{A}}(p, \epsilon; v, T)$ is an eigenvalue of the matrix $\mathbf{M}_{\mathrm{A}}$ [13] with a negative imaginary part, required by the boundary conditions on the retarded and advanced propagators. Note that equation (8) cannot be expanded in powers of v due to the essential singularity at $v = 0$. A consideration of equation (8) and the related equation for $x < 0$ indicates that exponential growth of the solution equation (7) induced by the $\otimes$ product is avoided only if the following stability conditions are respected [17]:

$$-\pi T < 2v\ \mathrm{Im}\ k_{\mathrm{A}} < 0 < 2v\ \mathrm{Im}\ k_{\mathrm{B}} < \pi T. \tag{9}$$

These stability conditions result in the introduction of two critical velocities v_{c1} and v_{c2} into the problem. They determine the boundaries of the region of stability. Thus for velocities smaller than v_{c1} (or larger than v_{c2}), the inequalities of equation (9) *are respected for all p, ϵ and no exponentially growing pieces of the time-ordered* Green's function are present. It may be verified (numerically) that high-energy quasiparticles at grazing incidence determine the boundary of the region of instability. It is natural to assume the existence of a high-energy cut-off ϵ_c that is the largest energy that contributes to equation (9) because such high-energy quasiparticles cannot possibly sense any difference between A and B symmetry in the order parameter [20]. The physical nature of the high-energy cut-off ϵ_c can, in principle, only be understood from a more sophisticated model of the order parameter. For example, the standard weak-coupling model employed in the present work ignores collisional broadening of the lifetime of Bogoliubov quasiparticles. Nevertheless, provided ϵ_c is not infinite it follows that $v_{c1}(T)$ is non-zero [20] and a knowledge of v_{c1} at a known temperature is sufficient to determine it at all temperatures:

$$v_{c1}(T_1) = (T_1/T_2)v_{c1}(T_2). \tag{10}$$

Note that an implication of this relation is that at zero temperature, the A–B phase boundary can never move rigidly.

As the measured terminal velocity $v_{\mathrm{AB}}(T)$ of the interface increases upon reduction of the temperature T one might attribute the dramatic signal changes observed by Boyd and Swift [3] at lower temperatures to the crossing of $v_{\mathrm{AB}}(T)$

with $v_{c1}(T)$. Since the order parameter is self-consistently determined from the fermionic distribution function this implies an instability of the moving kink in the order parameter to rigid motion.

It was first pointed out by Leggett and Yip [4] that the rather high terminal velocity of the A–B interface, observed when the B phase expands into the undercooled A phase, may be explained in terms of Andreev scattering of spin-$\frac{1}{2}$ excitations incident onto the moving interface, thereby giving rise to a friction force. In the absence of a temperature gradient across the interface, $T_A = T_B$, our exact result equation (7) suggests, at low propagation speeds $v < v_{c1}$, a frictionless and rigidly translating interface in the ballistic limit. This view seems to agree with general hydrodynamical considerations of Liu [21], who finds that it is exclusively the Kapitza resistance that governs the dissipation in the *moving* A–B phase boundary problem.

Such a scenario is, in the ballistic limit and for equal temperature on either side of the interface, quite distinct from the picture of a moving interface subject to a friction force caused by Andreev scattering at arbitrary propagation speed. The latter was envisaged by several authors [4, 18, 19], who also worked in the ballistic limit. We note that all of these authors assumed the possibility expanding in powers of v/v_F, however our explicit result equation (7) indicates the presence of an essential singularity of the form $\exp[-x\epsilon/v]$, which will be missed by any perturbative expansion in powers of v/v_F.

When $T_A \neq T_B$, our calculation leads to a non-equilibrium quasiparticle distribution which, in the ballistic limit, relaxes towards, equilibrium by quasiparticle collisions occuring predominantly near the container walls. It is these processes which, in our opinion, produce entropy in the ballistic limit and lead, eventually, to 'friction' in the moving A–B interface problem [22].

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Schopohl N and Waxman D 1989 *AIP Conf. Proc.* **194** 55 ed G G Ihas and Y Takano

[7] The vectors $\hat{l}$ and $\hat{d}$ of the A phase and the vector $\hat{n}$ of the B phase are oriented as described in [2]. Carets above vectors denote unit vectors throughout and $\theta(x)$ denotes the Heaviside step function

[8] The origin of non-unitary components of the gap tensor can be explained in terms of the spin dependence of the Andreev scattering. This spin dependence is characteristic of spin-$\frac{1}{2}$ excitations moving in a triplet order parameter with sharp spatial structure on the scale of ξ_{BCS}, e.g. near the A–B interface, inside the core of vortices or near a wall. Starting with the exact quasiparticle propagator calculated for a piecewise constant (and unitary!) order parameter the first iteration of the gap equations already displays *all* qualitative features of the full self-consistent solution, including non-unitary order-parameter components occuring there where the gap changes rapidly

[9] The Wigner transformation generalizes the concept of Fourier transformations to non-translational invariant functions. The $\otimes$ product between Wigner transforms is given by:

$$\mathbf{A}_{\mathrm{W}}(\boldsymbol{R},t;\boldsymbol{p},\epsilon)\otimes\mathbf{B}_{\mathrm{W}}(\boldsymbol{R},t;\boldsymbol{p},\epsilon)=\mathbf{A}_{\mathrm{W}}(\boldsymbol{R},t;\boldsymbol{p},\epsilon)$$
$$\times\exp\left[(1/2\mathrm{i})\left(\overleftarrow{\partial}_{\mathrm{p}}\vec{\partial}_{\mathrm{R}}-\overleftarrow{\partial}_{\mathrm{R}}\vec{\partial}_{\mathrm{p}}+\overleftarrow{\partial}_{t}\vec{\partial}_{\epsilon}-\overleftarrow{\partial}_{\epsilon}\vec{\partial}_{t}\right)\right]\mathbf{B}_{\mathrm{W}}(\boldsymbol{R},t;\boldsymbol{p},\epsilon)$$

Arrows on top of a derivative operator indicate if the operator acts to the left or to the right.
Ring P and Schuck P 1980 *The Nuclear Many-Body Problem* (Berlin: Springer)
Serene J W and Rainer D 1983 *Phys. Rep.* **101** 221
Klauder J R 1984 *Phys. Rev.* A **29** 2036

[10] Use of notation: σ_x, σ_y, σ_z are the Pauli matrices, σ_0 denotes the 2x2 identity matrix, and

$$\hat{\tau}_0=\begin{pmatrix}\sigma_0 & 0\\ 0 & \sigma_0\end{pmatrix}\qquad \hat{\tau}_1=\begin{pmatrix}0 & \sigma_0\\ \sigma_0 & 0\end{pmatrix}$$
$$\hat{\tau}_2=\begin{pmatrix}0 & -\mathrm{i}\sigma_0\\ \mathrm{i}\sigma_0 & 0\end{pmatrix}\qquad \hat{\tau}_3=\begin{pmatrix}\sigma_0 & 0\\ 0 & -\sigma_0\end{pmatrix}.$$

Our definition of the Green's function $G_{\mathrm{W}}(\boldsymbol{R},t;\boldsymbol{p},\epsilon)$ includes multiplication by

$$\begin{pmatrix}\sigma_0 & 0\\ 0 & -\mathrm{i}\sigma_y\end{pmatrix}$$

from the left and

$$\begin{pmatrix}\sigma_0 & 0\\ 0 & \mathrm{i}\sigma_y\end{pmatrix}$$

from the right to eliminate $\mathrm{i}\sigma_y$ factors;
Volovik G E 1990 *Physica* B **162** 222

[11] Momenta are measured in units of the Fermi momentum p_{F}, distances in units of the inverse Fermi momentum $1/p_{\mathrm{F}}$, energy in units of $p_{\mathrm{F}}^2/2m$ and velocity in units of the Fermi velocity p_{F}/m.

[12] The generalization to a smooth order-parameter profile simply entails the replacement of the exponentials in equation (4) by the x-ordered exponential:

$$T_x\exp\left\{2\mathrm{i}\int_0^x \mathbf{M}(x')\mathrm{d}x'\right\}$$

Details of the derivation will be presented elsewhere.

[13] The inverse of $(z - \mathbf{M})$ (see equation $(5a)$) may be given in the following 4×4 block form:

$$1/(z - \mathbf{M}) = \begin{pmatrix} \frac{1}{(z+\mathbf{a})^2 - H}(z_{\mathbf{a}} + \mathbf{a}) & \frac{1}{(z+\mathbf{a})^2 - \mathbf{H}} \\ -\hat{\tau}_0 + (z + \mathbf{a})\frac{1}{(z+\mathbf{a})^2 - \mathbf{H}}(z + \mathbf{a}) & (z + \mathbf{a})\frac{1}{(z+\mathbf{a})^2 - \mathbf{H}} \end{pmatrix}$$

There are, in general, eight different poles of this matrix in the complex z-plane determining the eigenvalues of $\mathbf{M}$. For a *unitary* order parameter, $\boldsymbol{\Delta} \times i\boldsymbol{\Delta}^* = O$, *the eigenvalues are twofold* degenerate and their position in the complex z plane is determined by solving for the zeroes of a *quartic* polynomial in z: $[(z + p_x)^2 + p_y^2 + p_z^2 - \mu]^2 + \boldsymbol{\Delta} \cdot \boldsymbol{\Delta}^* - [\epsilon - 2v(z + p_x)]^2 = 0$.

[14] If the velocity v of the interface reaches the spin-wave velocity $v_{\rm spin}$, there should occur radiation of shock waves caused by coupling of spin degrees of freedom to order-parameter collective modes;

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Nozières private communication

[17] These *stability conditions are not an artefact of the stratified-medium ansatz* for the order parameter. Their validity for a smooth order-parameter profile that translates rigidly at constant speed v may be proved inserting into equation (7) the exact solution of equation (3) expressed in terms of the x-ordered exponential [12]

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[19] Kopnin N B 1987 *Sov. Phys.-JETP* **65** 1187

[20] At the present time we have no knowledge of the high energy cut-off ϵ_c and believe it cannot be larger than the BCS cut-off. An estimate of v_{c1} and v_{c2} may be analytically obtained from equation (9) by setting the gap parameter $\Delta = 0$ and $\epsilon = \epsilon_c$: $v_{c1} = (\pi/2)(T/\sqrt{E_F\epsilon_c})v_F$ and $v_{c2} = \sqrt{\epsilon_c/E_F}\,v_F$, the high speed v_{c2} representing an unphysical solution branch of the non-linear conditions equation (9).

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[22] There exists substantial enhancement of particle–hole asymmetry in the moving A–B interface problem caused by the $\boldsymbol{p}' \cdot \dot{\boldsymbol{R}}_0$ term in equation (3). We find a non-trivial effect of velocity on the bound-state spectrum of the fermions localized in the direction perpendicular to the A–B interface [6]. For a finite difference in thermodynamical potential across the interface, $\Omega_{\rm A} \neq \Omega_{\rm B}$, the interface will accelerate. In the ballistic regime the terminal velocity is limited by dissipative processes occuring during the rapid adiabatic passage of the interface, e.g. pair breaking [4], but also Zener tunnelling of spin-$\frac{1}{2}$ excitations and transitions between the levels of the bound states.